ME 321: Fluid Mechanics-I

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Lecture - 02 (19/04/2025)
Fluid Dynamics: Basic concepts

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Important numbers in Fluid Mechanics

1. Reynolds number, Re
2. Mach number, M
3. Capillary number, Ca
4. Knudsen number, Kn

There are many other numbers which will dominate the fluid flow depending on the application of the system.

Reynolds number, Re

Inertial / inertia force, F_I of a fluid element that has a **length scale L** (characteristic dimension) can be determined as:

$$F_I = ma \propto \left(\rho L^3\right)\left(\frac{L}{T^2}\right) = \left(\rho L^2\right)\left(\frac{L}{T}\right)^2 = \rho L^2 V^2$$

Viscous force, F_V can be determined as:

$$F_V = \mu \frac{dV}{dy} A \propto \mu \left(\frac{V}{L}\right)\left(L^2\right) = \mu V L$$

Reynolds number, Re is a measure of the ratio of inertia to the viscous force in a fluid flow system having a velocity scale V, length scale L, and fluid properties - density, ρ and dynamic viscosity, μ (or kinematic viscosity, $\nu = \mu/\rho$)

$$\mathrm{Re} = \frac{\text{inertial force}}{\text{viscous force}} = \frac{F_I}{F_V} = \frac{\rho V^2 L^2}{\mu V L}$$

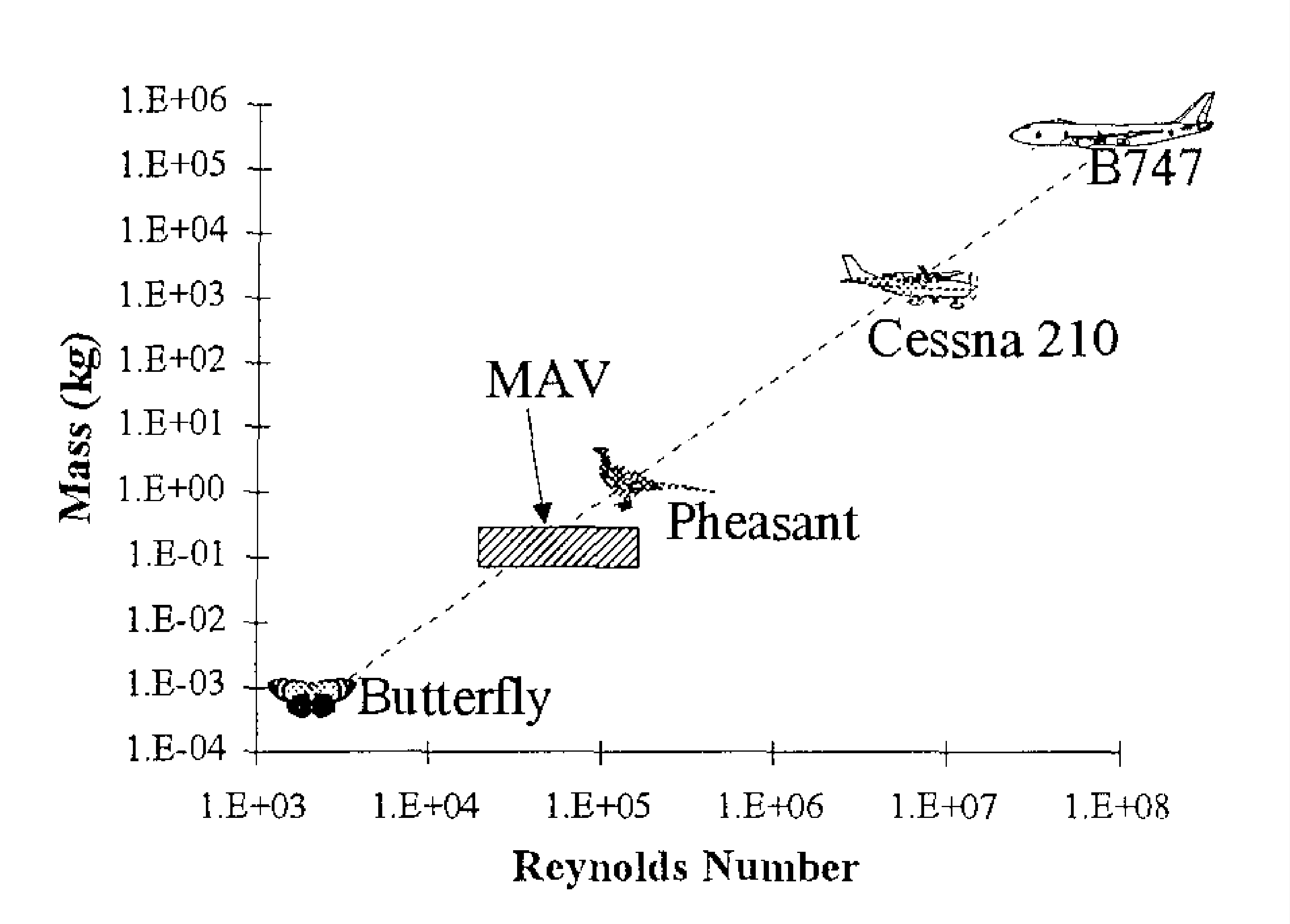
$$\Rightarrow \quad \mathrm{Re} = \frac{\rho V L}{\mu}$$

$$\boxed{Re = \frac{VL}{\nu}}$$

Re is important for almost all types of flow.

Gives the basic idea on the nature of flow: laminar or turbulent

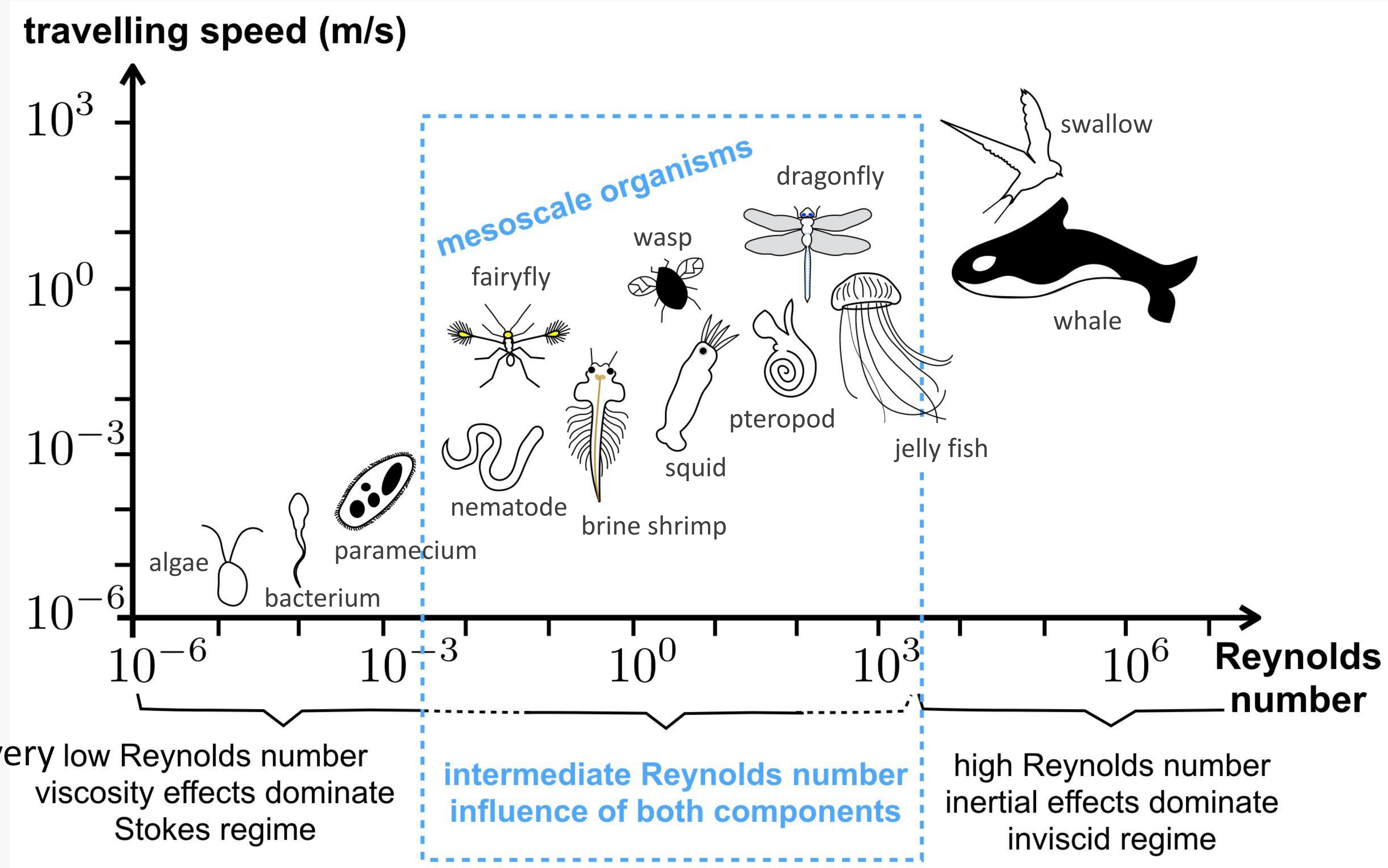
Reynolds number, Re



$$Re = \frac{VL}{\nu}$$

T. J. Mueller, Aerodynamic Measurements at Low Raynolds Numbers for Fixed Wing Micro-Air Vehicles, ADPO10760 (1999).

Reynolds number, Re



$$Re = \frac{VL}{\nu}$$

Reynolds numbers in the animal kingdom, highlighting the intermediate Reynolds range for swimming and flying organisms. The organisms shown schematically from low to high Reynolds numbers

D. Klotsa, As above, so below, and also in between: mesoscale active matter in fluids, Soft Matter, 2019.

Mach number, M / Ma

Elastic force, F_E on a fluid element that has a **length scale L** with **bulk modulus of elasticity B** is given by

$$F_E = BA \propto BL^2$$

Mach number, M is a measure of the ratio of inertial to the elastic force in a fluid flow system having a velocity scale V, length scale L and bulk modulus of elasticity B.

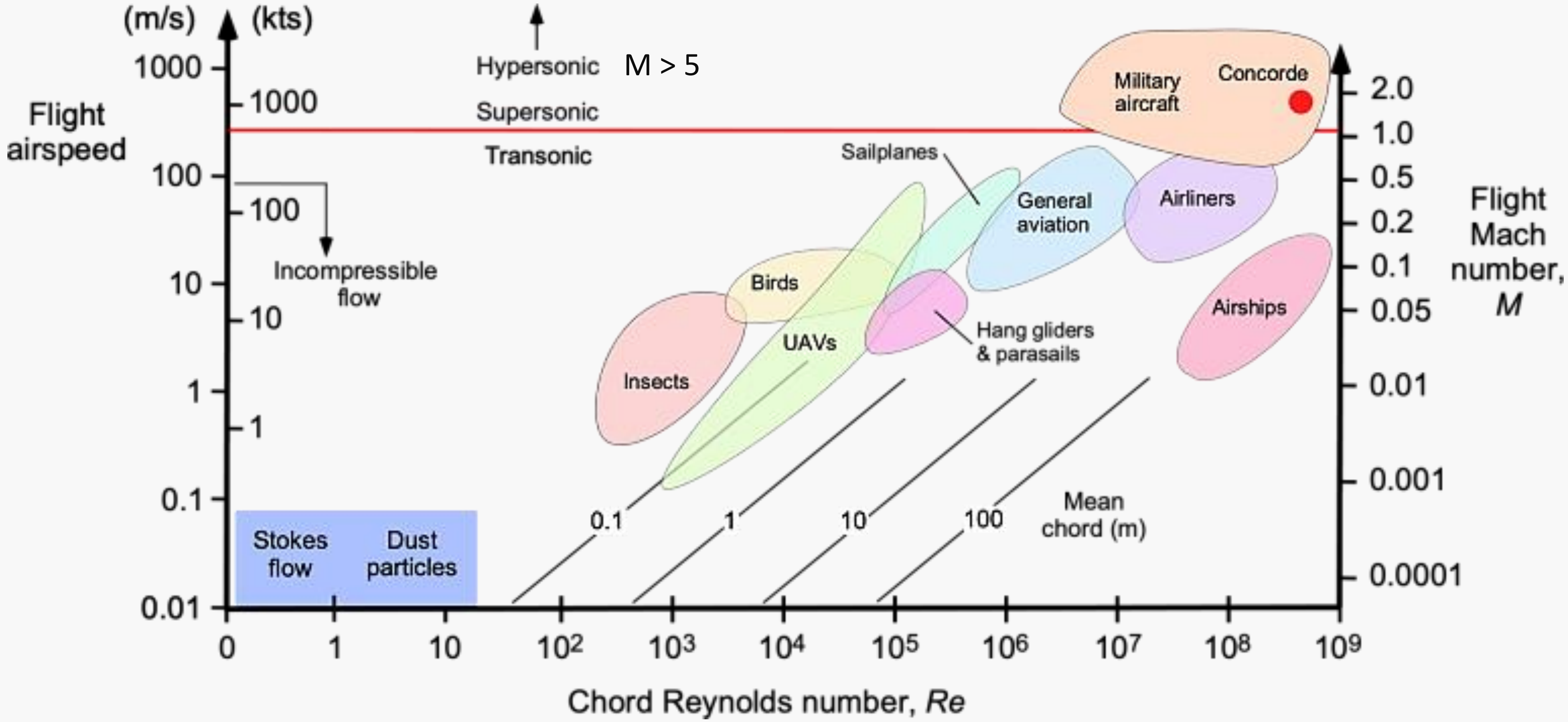
$$\mathrm{M}^2 = \frac{\text{inertial force}}{\text{elastic force}} = \frac{F_I}{F_E} = \frac{\rho V^2 L^2}{BL^2}$$

$$\Rightarrow \quad \mathrm{M}^2 = \frac{\rho V^2}{B} = \frac{V^2}{\left(\frac{B}{\rho}\right)} = \frac{V^2}{c^2} \quad ; c = \text{speed of sound at that medium}$$

$$\Rightarrow \quad \mathrm{M} = \frac{V}{c}$$

M is important for high speed flows where compressibility effects can't be neglected.

Mach number, M / Ma



Range of Reynolds and Mach numbers encountered by various flight articles

https://eaglepubs.erau.edu/introductiontoaerospaceflightvehicles/chapter/mach-number-and-reynolds-number/

Capillary number, Ca

The ***capillary number*** is a dimensionless quantity that relates the **viscous** force in a fluid flow system to the **surface tension** forces. It is defined as

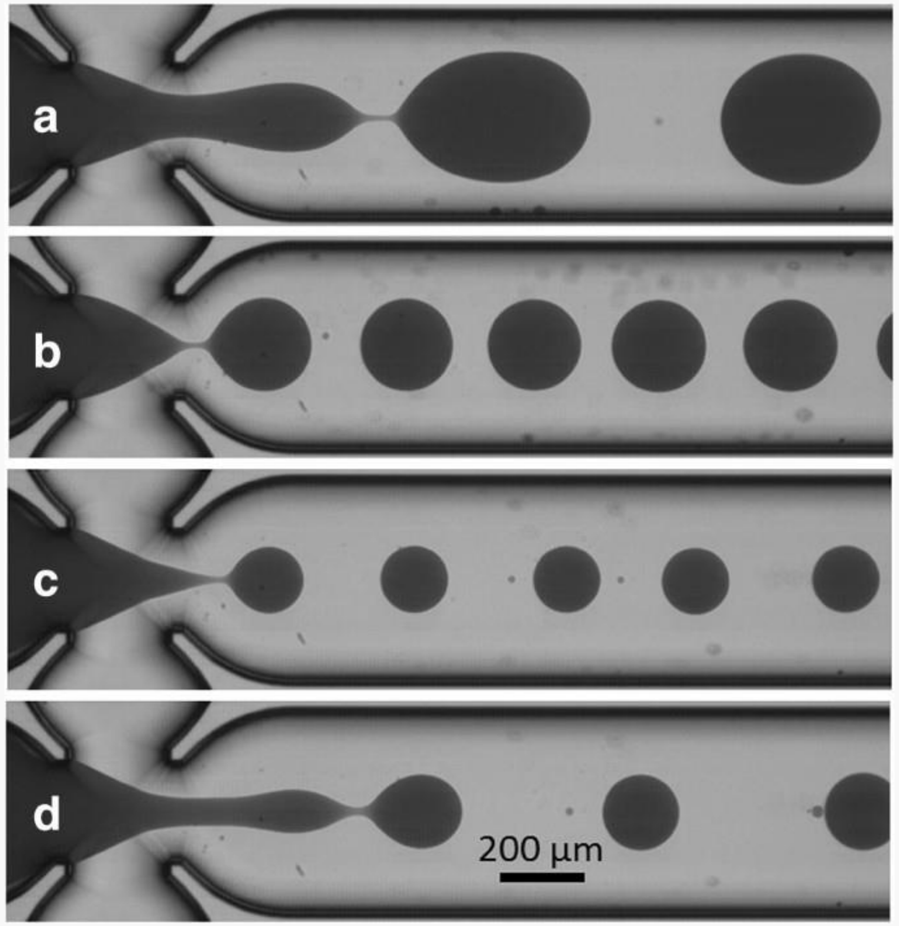
$$Ca = \frac{\text{viscous force}}{\text{Surface tension}} = \frac{\mu V}{\sigma}$$

μ = dynamic viscosity of fluid, V = characteristic velocity scale and σ = surface tension

The capillary number (Ca) is used whenever the forces resulting from fluid motion are to be compared to the forces resulting from surface tension (interfacial phenomena).

This is the case if a liquid is moved across a second fluid layer, *e.g.*, a gas or an immiscible second liquid.

A good visual example for these effects are droplets suspended in an inert liquid in **droplet microfluidics**.

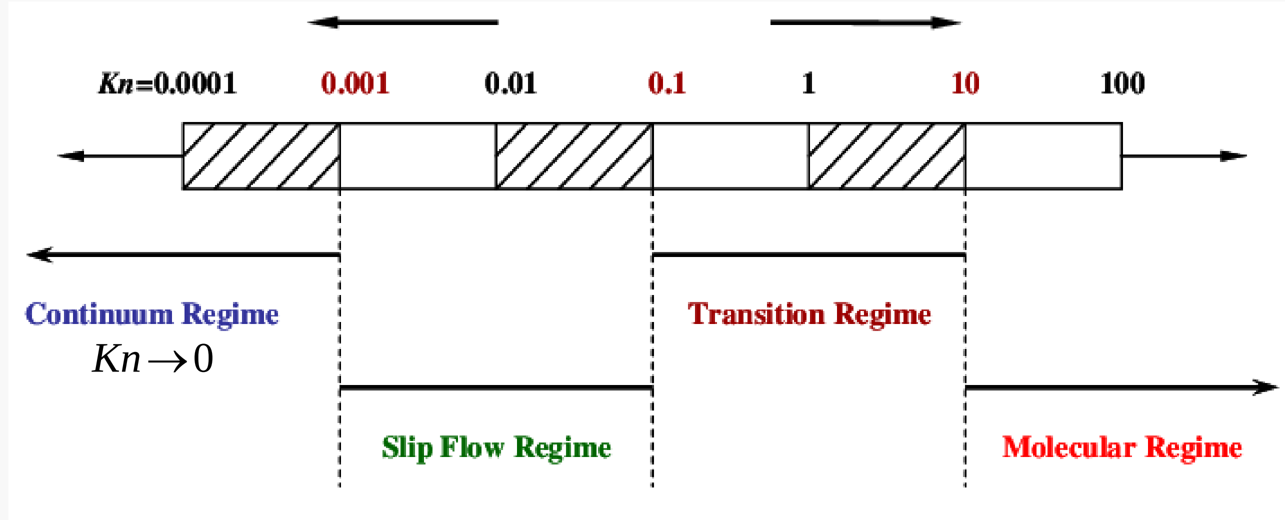


Kovalchuk, N.M., Sagisaka, M., Steponavicius, K. *et al.* Drop formation in microfluidic cross-junction: jetting to dripping to jetting transition. *Microfluid Nanofluid* **23**, 103 (2019)

Flow classification based on molecular action

Table: Flow classification

Regime	*kn*
Continuum flow	$kn < 0.001$
Slip flow	$0.001 < kn < 0.1$
Transitional flow	$0.1 < kn < 10$
Free molecular flow	$kn > 10$



$$\text{Knudsen number, } Kn = \frac{\lambda}{L}$$

$$Kn = \frac{\text{mean free path of gas}}{\text{characteristic dimension}}$$

(**Continuum approach:** individual molecule is not important rather averaged behavior is considered in bulk; continuous flow) (Undergraduate level)

Classical fluid mechanics

(**Molecular approach:** individual molecule is important and should be modeled; gas kinetic theory, statistical approach, etc.) (Advanced graduate level)

Micro/nano scale fluid mechanics, rarefied gas dynamics, space applications, etc.

A larger Knudsen number (Kn) arises either due to **longer mean free path (rarefied gas flows)** or **small characteristic dimension of the system (microsystems).**

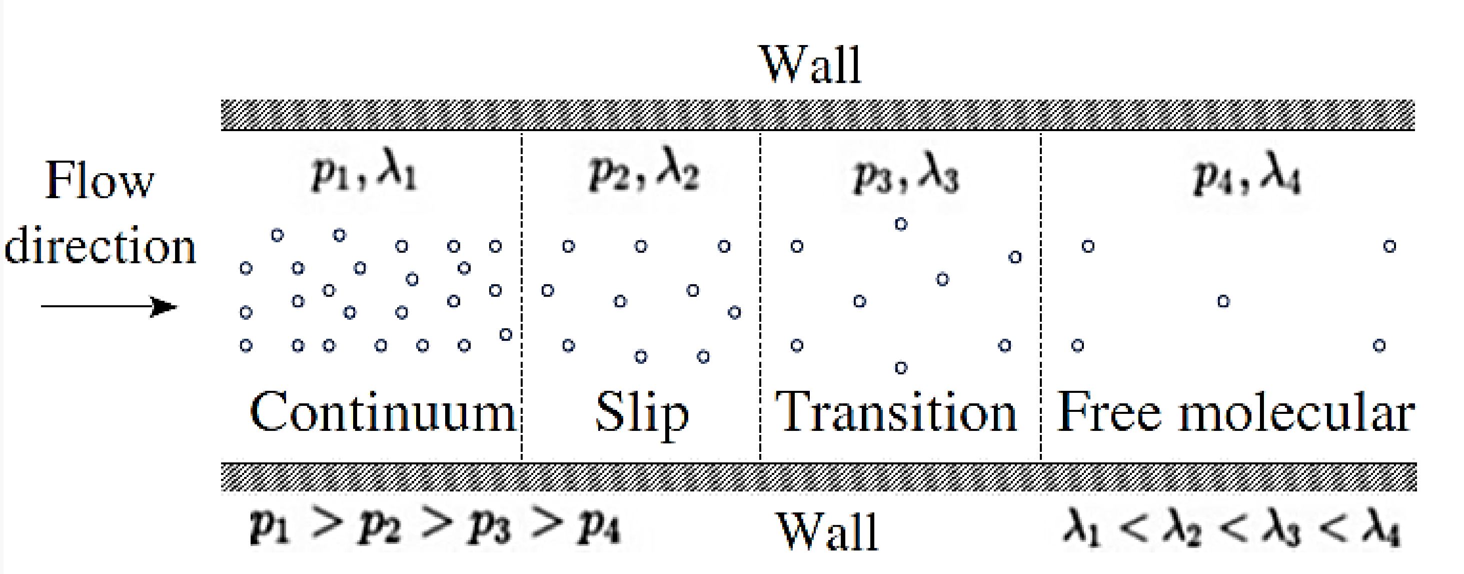


Fig. 1.3 With mean free path increasing due to reduction in pressure, the flow transits through various regimes. (Here p denotes pressure and λ denotes mean free path and hollow circles represent the molecules in random motion)

For very low Knudsen number(Kn→0), the number of collisions between the molecules is large compared to the number of collisions between the molecules and the wall. In such a case, the usual continuum concept is applicable and the Navier–Stokes equations and the Fourier heat conduction law are valid. The **continuum flow** is characterized by the **Reynolds number (Re)** and the **Mach number (Ma)** only; the **Knudsen number will not enter** the problem explicitly since it is already considered to be very small.

$$\text{Knudsen number, } Kn = \frac{\lambda}{L}$$

*Agrawal et al. Microscale Flow and Heat Transfer Mathematical Modelling and Flow Physics (2020)

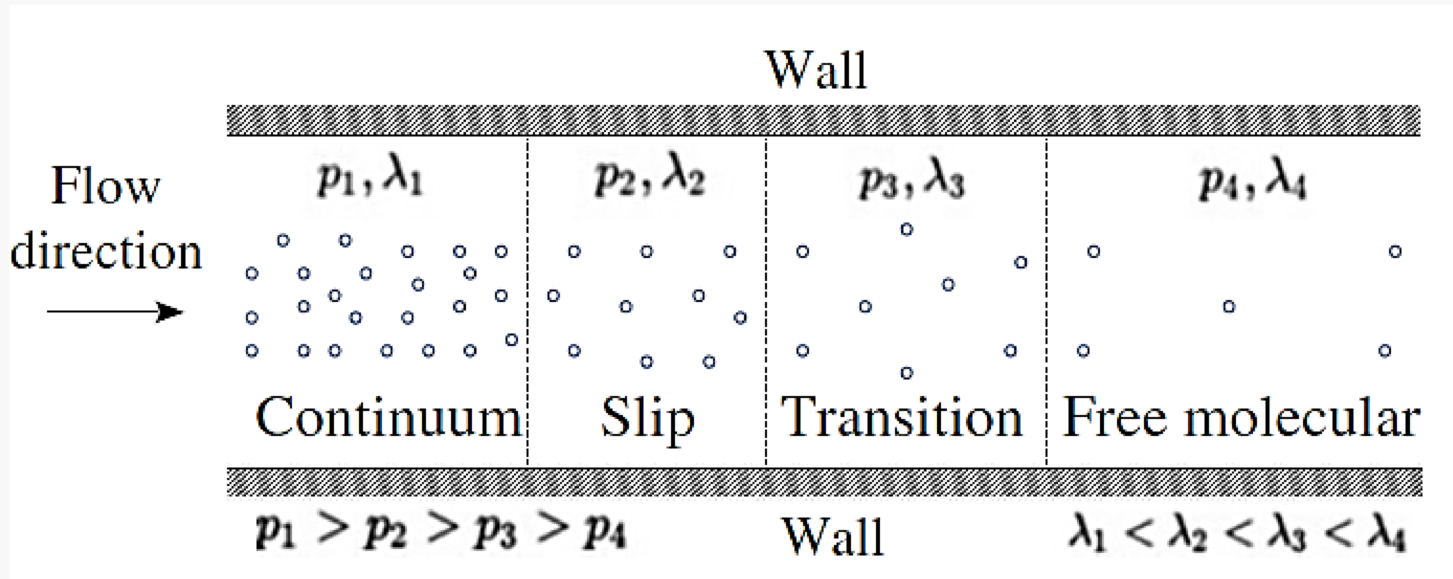


Fig. 1.3 With mean free path increasing due to reduction in pressure, the flow transits through various regimes. (Here p denotes pressure and λ denotes mean free path and hollow circles represent the molecules in random motion)

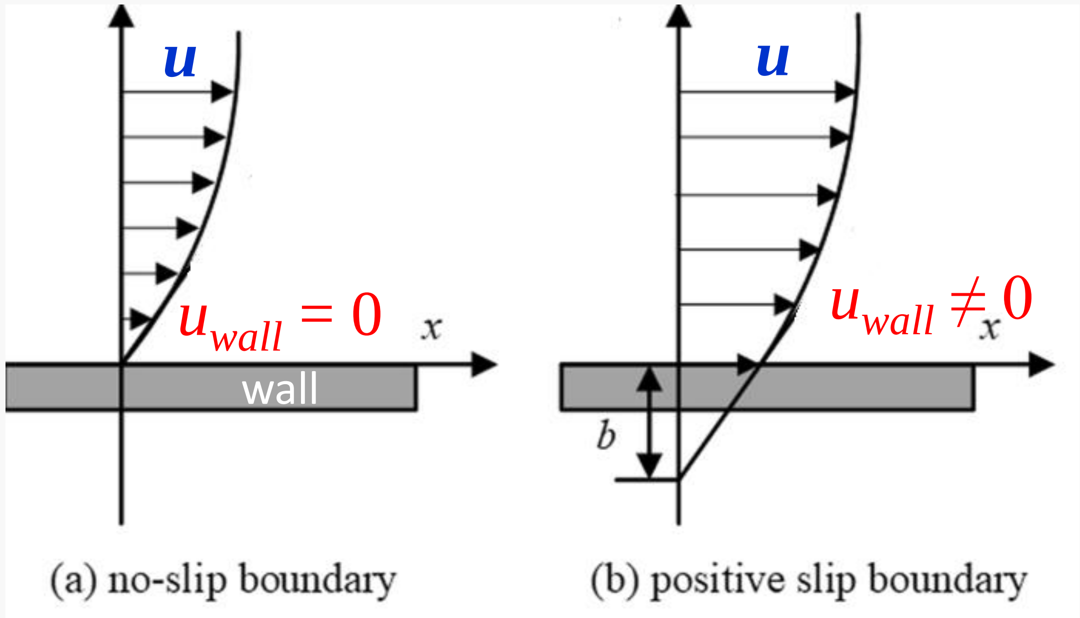
For sufficiently large Knudsen number, the continuum concept needs to be modified to predict the fluid flow and heat transfer characteristics. In such a case, **first- (or second-) order slip boundary condition needs to be considered in the slip and early transition regimes**.

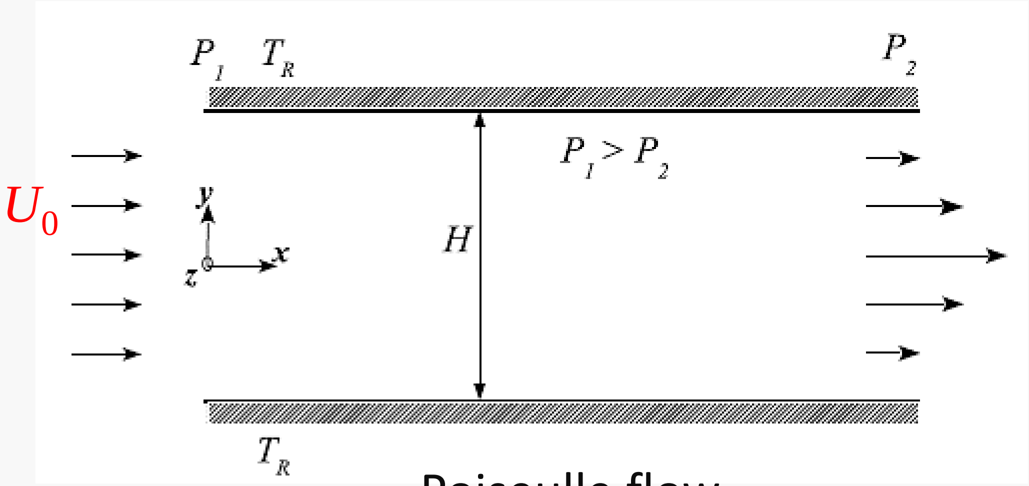
$$\text{Knudsen number, } Kn = \frac{\lambda}{L}$$

At higher Knudsen numbers, higher-order continuum transport equations need to be considered to analyze the fluid flow and heat transfer behavior **(Boltzmann equation, DSMC/MD)**.

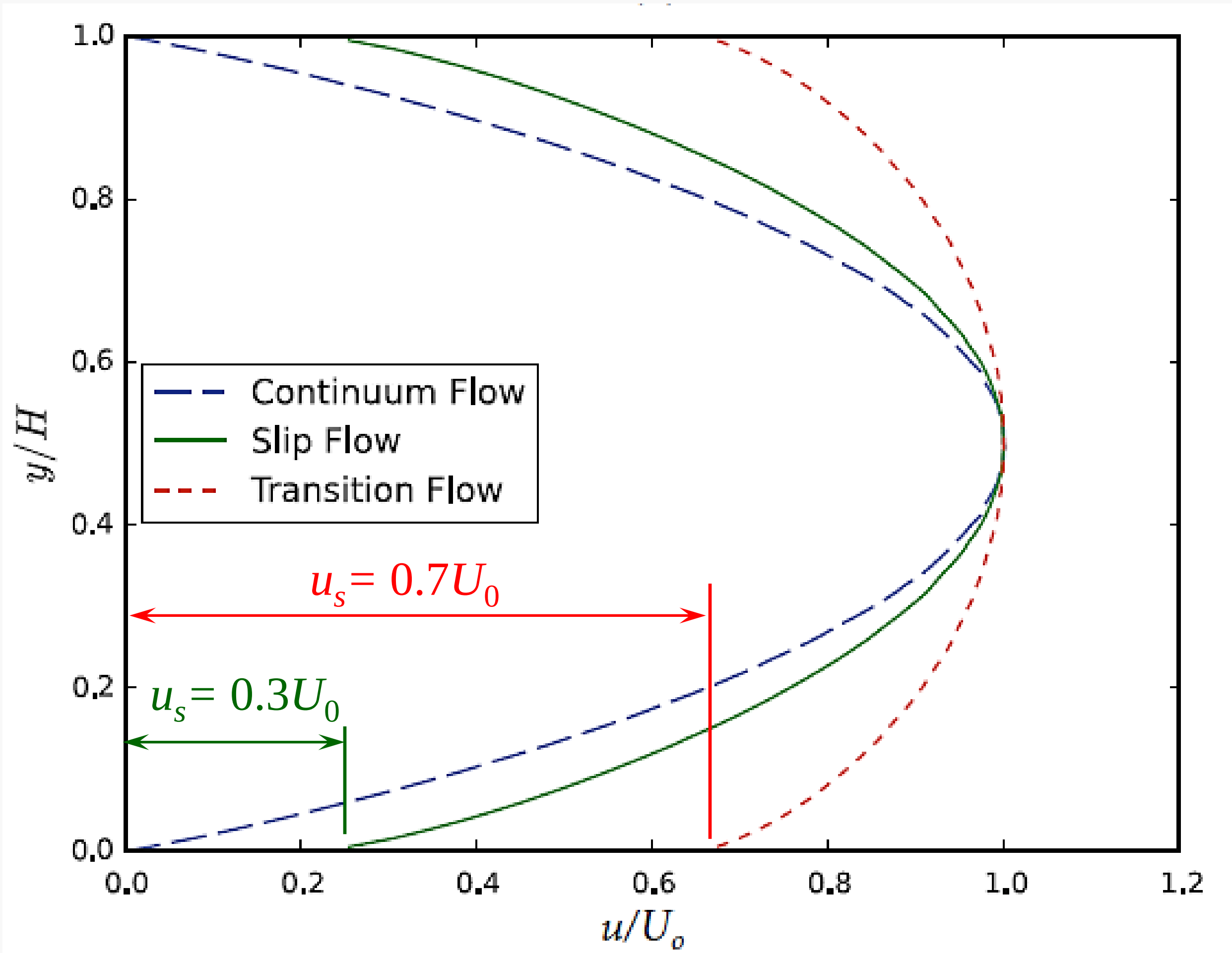
*Agrawal et al. Microscale Flow and Heat Transfer Mathematical Modelling and Flow Physics (2020)

Velocity slip





Poiseulle flow



*Agrawal et al. Microscale Flow and Heat Transfer Mathematical Modelling and Flow Physics (2020)

Molecular approach

There are borderline cases for gases at such low pressures that molecular spacing and mean free path are comparable to, or larger than, the physical size of the system **(rarefied flows)**. Molecular approach is required.

Table A.6 Properties of the Standard Atmosphere

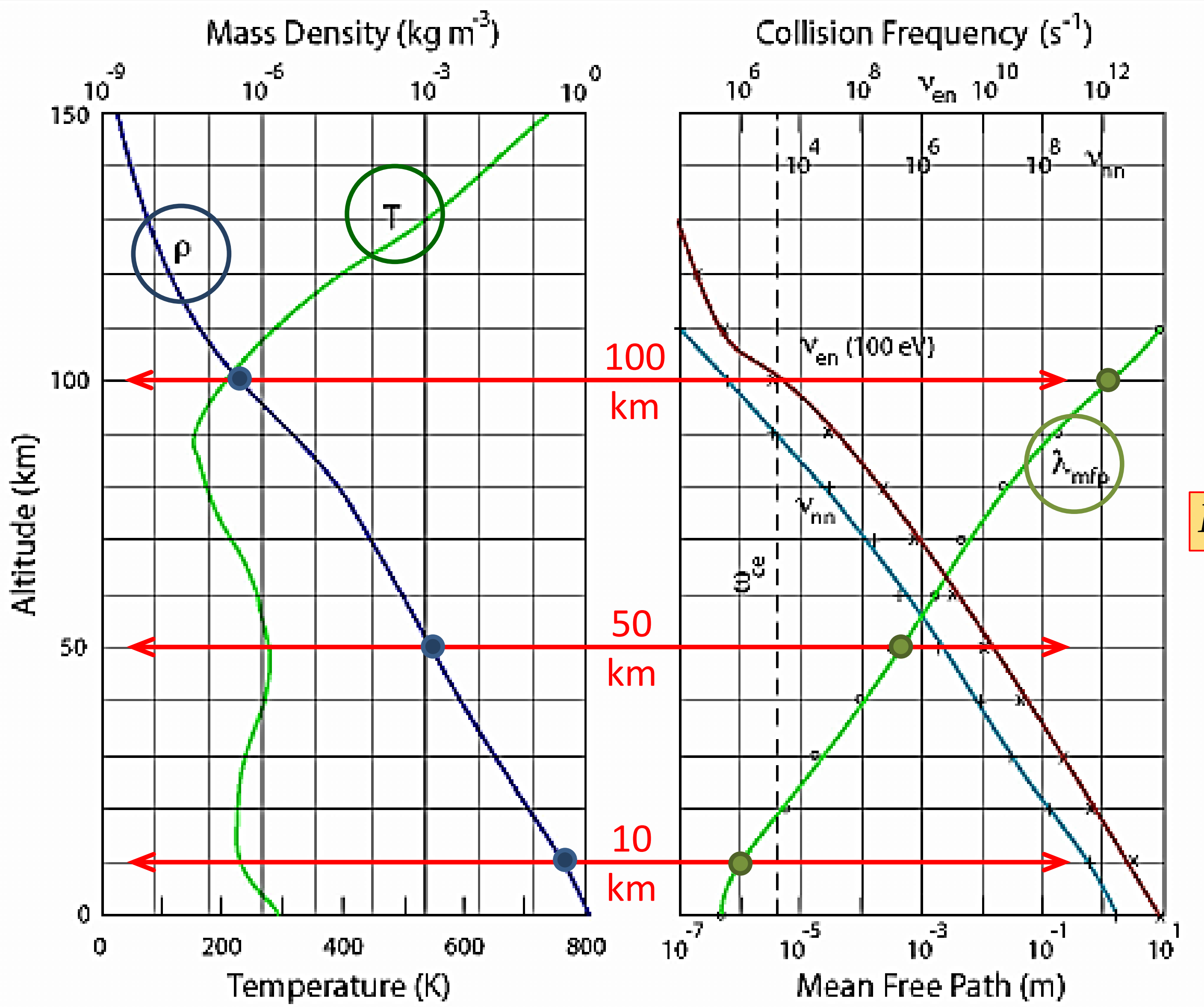
z, m	T, K	p, Pa	ρ, kg/m^3	a, m/s
−500	291.41	107,508	1.2854	342.2
0	288.16	101,350	1.2255	340.3
500	284.91	95,480	1.1677	338.4
1000	281.66	89,889	1.1120	336.5
1500	278.41	84,565	1.0583	334.5
2000	275.16	79,500	1.0067	332.6
2500	271.91	74,684	0.9570	330.6
3000	268.66	70,107	0.9092	328.6
3500	265.41	65,759	0.8633	326.6
4000	262.16	61,633	0.8191	324.6
4500	258.91	57,718	0.7768	322.6
5000	255.66	54,008	0.7361	320.6
5500	252.41	50,493	0.6970	318.5
6000	249.16	47,166	0.6596	316.5
6500	245.91	44,018	0.6237	314.4
7000	242.66	41,043	0.5893	312.3
7500	239.41	38,233	0.5564	310.2
8000	236.16	35,581	0.5250	308.1
8500	232.91	33,080	0.4949	306.0
9000	229.66	30,723	0.4661	303.8
9500	226.41	28,504	0.4387	301.7
10,000	223.16	26,416	0.4125	299.5
10,500	219.91	24,455	0.3875	297.3
11,000	216.66	22,612	0.3637	295.1
11,500	216.66	20,897	0.3361	295.1
12,000	216.66	19,312	0.3106	295.1
12,500	216.66	17,847	0.2870	295.1
13,000	216.66	16,494	0.2652	295.1
13,500	216.66	15,243	0.2451	295.1
14,000	216.66	14,087	0.2265	295.1
14,500	216.66	13,018	0.2094	295.1
15,000	216.66	12,031	0.1935	295.1
15,500	216.66	11,118	0.1788	295.1
16,000	216.66	10,275	0.1652	295.1
16,500	216.66	9496	0.1527	295.1
17,000	216.66	8775	0.1411	295.1
17,500	216.66	8110	0.1304	295.1
18,000	216.66	7495	0.1205	295.1
18,500	216.66	6926	0.1114	295.1
19,000	216.66	6401	0.1029	295.1
19,500	216.66	5915	0.0951	295.1
20,000	216.66	5467	0.0879	295.1
22,000	218.60	4048	0.0645	296.4
24,000	220.60	2972	0.0469	297.8
26,000	222.50	2189	0.0343	299.1
28,000	224.50	1616	0.0251	300.4
30,000	226.50	1197	0.0184	301.7
40,000	250.40	287	0.0040	317.2
50,000	270.70	80	0.0010	329.9
60,000	255.70	22	0.0003	320.6
70,000	219.70	6	0.0001	297.2

Mean free path of atmosphere	
Sea level	$\lambda \sim 0.1\ \mu m$
z = 50 km	$\lambda \sim 0.1$ mm
z = 150 km	$\lambda \sim 1$ m

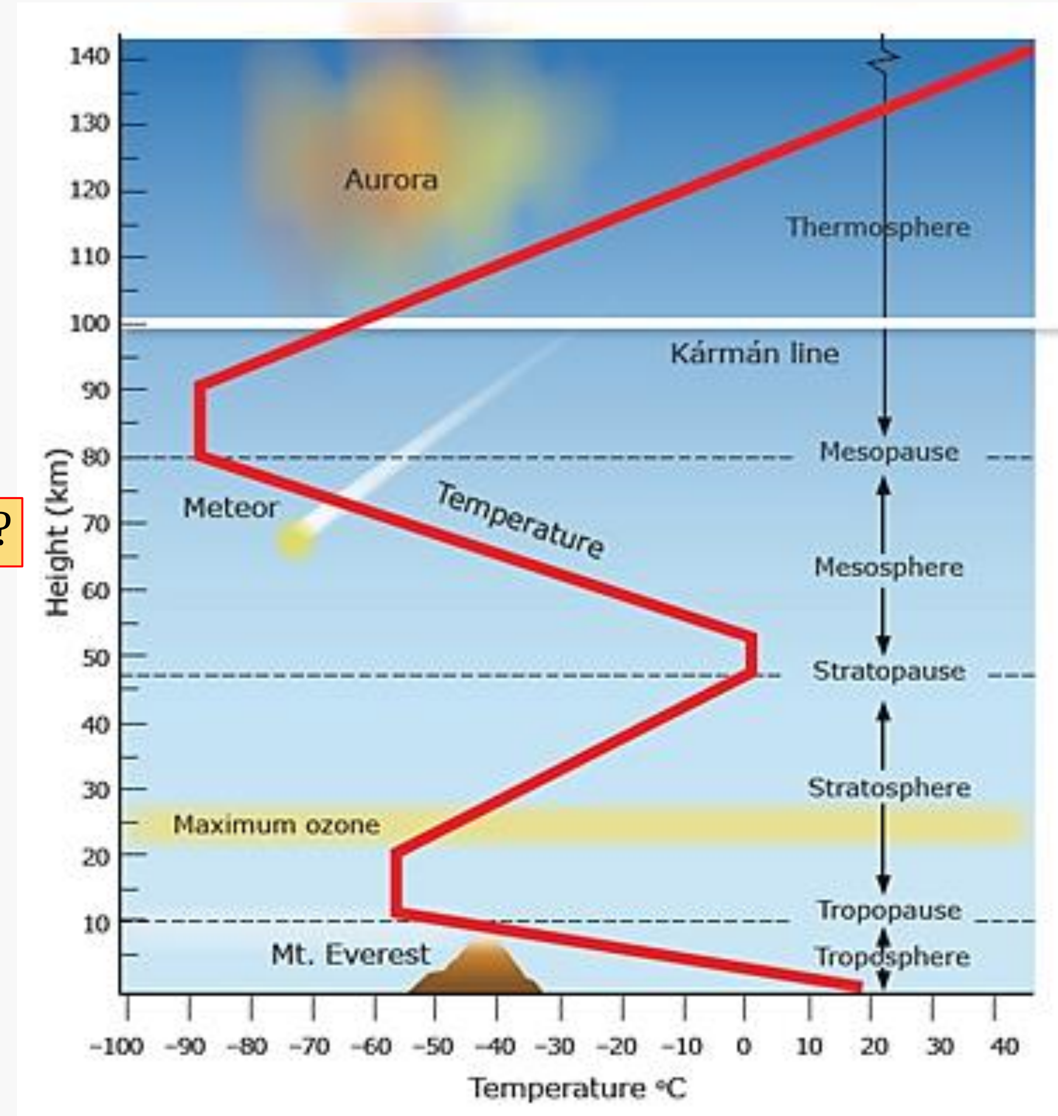
$$\text{Knudsen number, } Kn = \frac{\lambda}{L}$$

$$Kn = \frac{\text{mean free path of gas}}{\text{characteristic dimension}}$$

Atmosphere & Mean free path



Kn?



$$\text{microscale,} \quad L\downarrow \quad \mathrm{Re}\downarrow$$

$$\text{high speed flow,} \quad V\uparrow \quad (M>1) \quad \mathrm{Re}\uparrow$$

$$\text{Reynolds number}, \mathrm{Re} = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho V L}{\mu}$$

$$\text{Mach number}, M = \frac{\text{Flow velocity}}{\text{Sound velocity}} = \frac{V}{a}$$

$$\text{high speed high-altitude flow,} \quad p\downarrow \rho\downarrow \quad V\uparrow \quad (M>1) \quad \mathrm{Re}\downarrow$$

(Low density low Re supersonic/hypersonic flows at very high altitude from earth's sea-level)

Problem

Mean free path of gas can be determined by:

$$\lambda = 1.26 \frac{\mu}{\rho \sqrt{(RT)}} = 1.26 \frac{\mu}{p} \sqrt{(RT)}$$

- Estimate the mean free path of air at 20°C and 7 kPa.
- What is the smallest length scale of a flow system that will ensure the validity of continuum hypothesis?

Regime	***kn***
Continuum flow	kn < 0.001
Slip flow	0.001< kn < 0.1
Transitional flow	0.1< kn < 10
Free molecular flow	kn > 10

Table 1.1 Classification of flow regimes and models

Knudsen number	Regime	Fluid model
$Kn < 10^{-3}$	Continuum	Navier–Stokes equations with no slip boundary conditions
$10^{-3} < Kn < 10^{-1}$	Slip flow	Navier–Stokes equations with velocity slip and temperature jump boundary conditions
$10^{-1} < Kn < 10$	Transition	Higher order continuum transport equations (e.g., Burnett, Grad, OBurnett, O13, etc.)
$Kn > 10$	Free molecular flow	Collisionless Boltzmann equation